Can we trust to the N-body simulations of dark matter structures?

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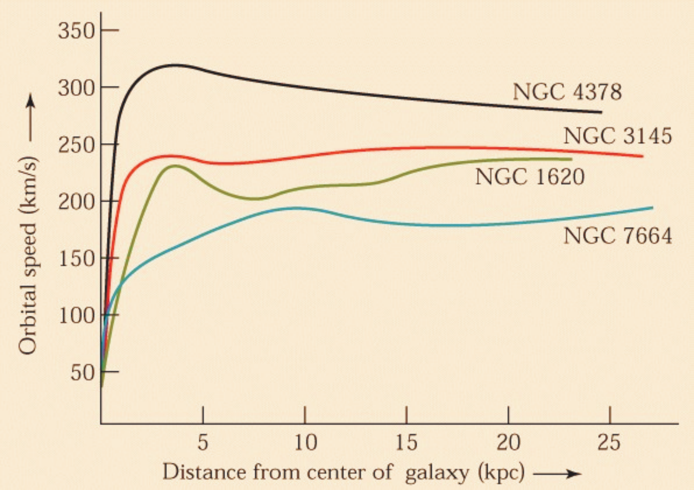
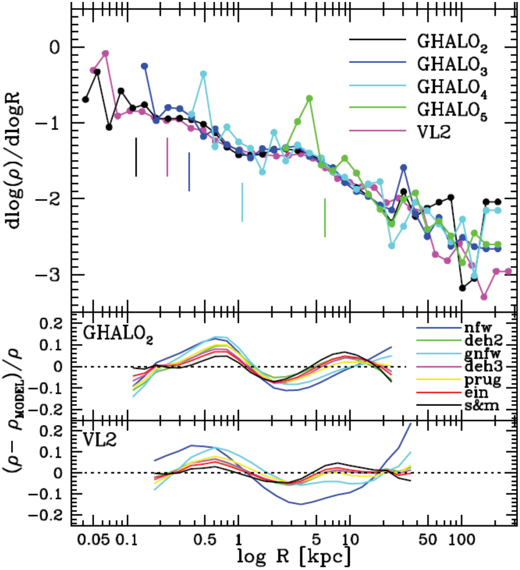
Dark matter. Discovery.



Density profiles. N-body simulations (Stadel et al. 2009)



Density profiles. Theory.

Isothermal profile

$$\rho \sim r^{-2}$$

Navarro-Frenk-White profile

$$\rho_{NFW} = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

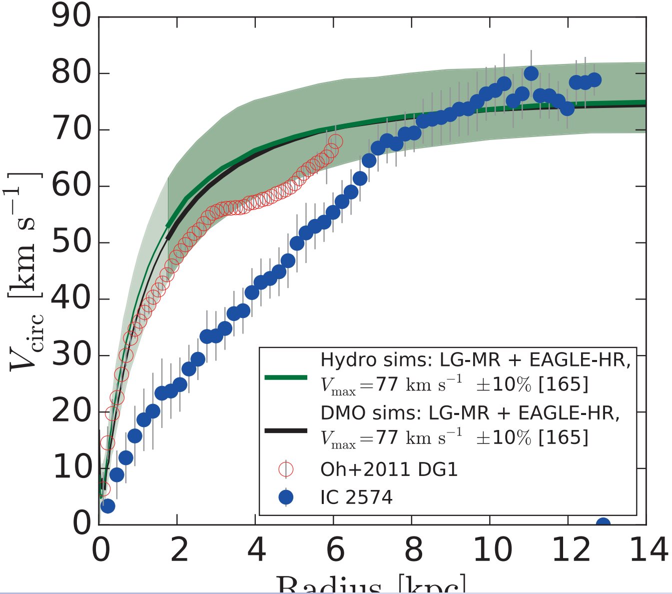
Einasto profile

$$\rho_{Ei} = \rho_s \exp\left\{-2n\left[\left(\frac{r}{r_s}\right)^{\frac{1}{n}} - 1\right]\right\}$$

Hernquist profile

$$\rho_H = \frac{Ma}{2\pi r(r + a)^3}$$

Simulations vs. observations (Oman et al. 2015)



Numerical effects

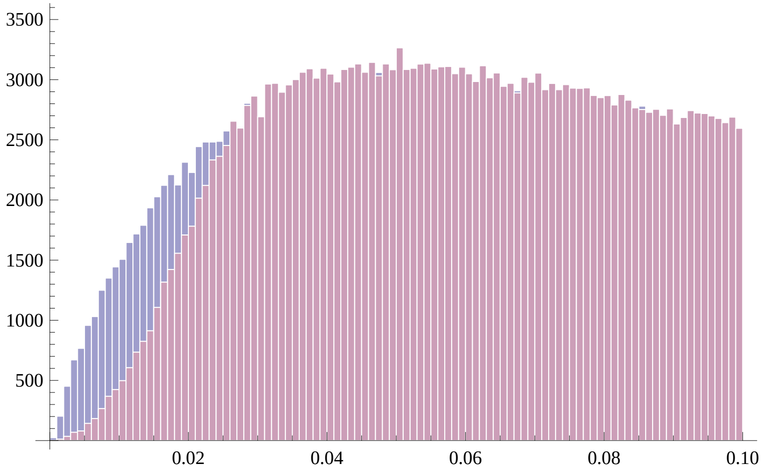
Relaxation time

$$\langle \Delta v \rangle \simeq 0 \qquad \langle \Delta v^2 \rangle \simeq \frac{8v^2 \ln \Lambda}{N(r)}$$

$$\tau_r(r) = \frac{N(r)}{8 \ln \Lambda} \cdot \tau_d(r) \qquad \tau_d(r) \sim \frac{r}{v}$$

(Power et. al. 2003) $t_0 \leq 1.7\tau_r$
(Hayashi et al. 2003; Klypin et al. 2013) $t_0 \leq 30\tau_r$

Core formation



Simulation details. Gadget-3.

$$\rho_H = \frac{Ma}{2\pi r(r+a)^3} \qquad \phi(r) = -\frac{GM}{r+a}$$

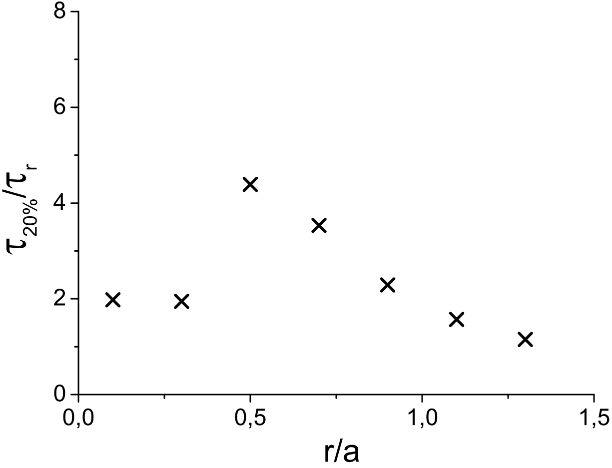
$M = 10^9 M_\odot$, $a = 100$ pc. We use $N = 10^6$ test bodies.

The relaxation time at $r = a$ is $\simeq 8.8 \cdot 10^{16}$s $\simeq 2.8 \cdot 10^9$ years. Therefore, we make 200 snapshots with the time interval $\Delta t = 10^{15}$ s, covering the time from 0 to $t_{max} = 2 \cdot 10^{17}$s $\simeq 6.5 \cdot 10^9$ years.

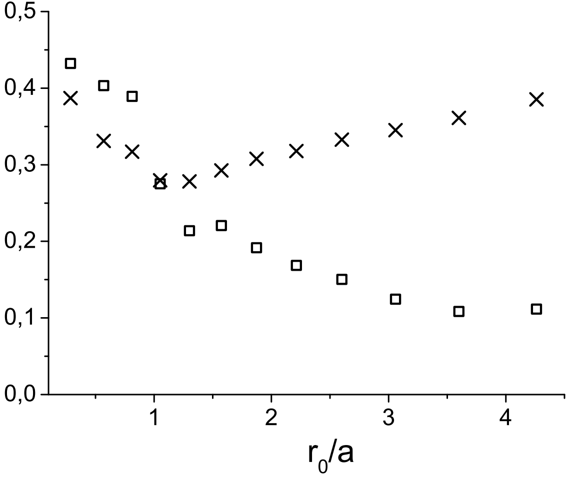
The integrals of motion $\epsilon = \phi(r) + v^2/2$, $\vec{K} = [\vec{v} \times \vec{r}]$, r_0:

$$\epsilon = \phi(r_0) + K^2/2r_0$$

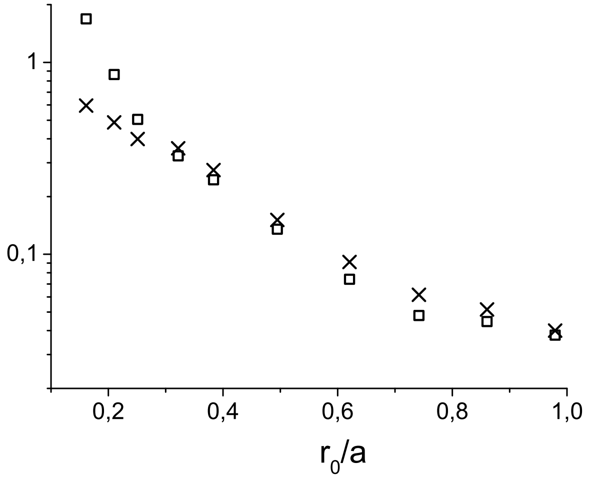
Core formation



$\langle \Delta K / K_{circ} \rangle$ (squares) and $\langle \Delta r_0 / r_0 \rangle$ (crosses)



The ratios $\frac{K_{circ}}{\tau_r}\left\langle\frac{\Delta K}{\Delta t}\right\rangle^{-1}$ (squares) and $\frac{1}{\tau_r}\left\langle\frac{\Delta r_0}{r_0\Delta t}\right\rangle^{-1}$ (crosses)



Kinetic equations

$$\frac{df}{dt} = \frac{\partial}{\partial p_\alpha}\left\{\tilde{A}_\alpha f + \frac{\partial}{\partial p_\beta}[B_{\alpha\beta} f]\right\}$$

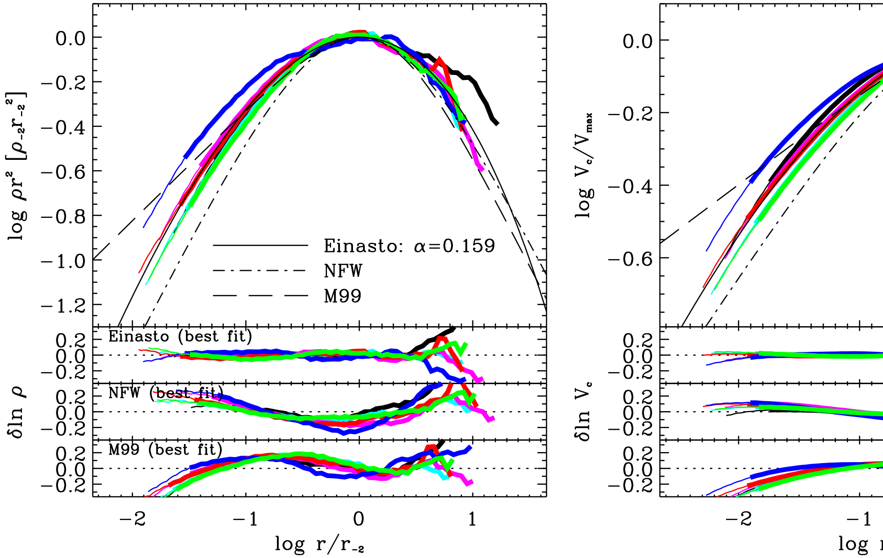
where $\vec{q}$ is the momentum changing $\vec{p} \to \vec{p} - \vec{q}$ in a unit time.

$$\tilde{A}_\alpha = \frac{\sum q_\alpha}{\delta t} \qquad B_{\alpha\beta} = \frac{\sum q_\alpha q_\beta}{2\delta t}$$

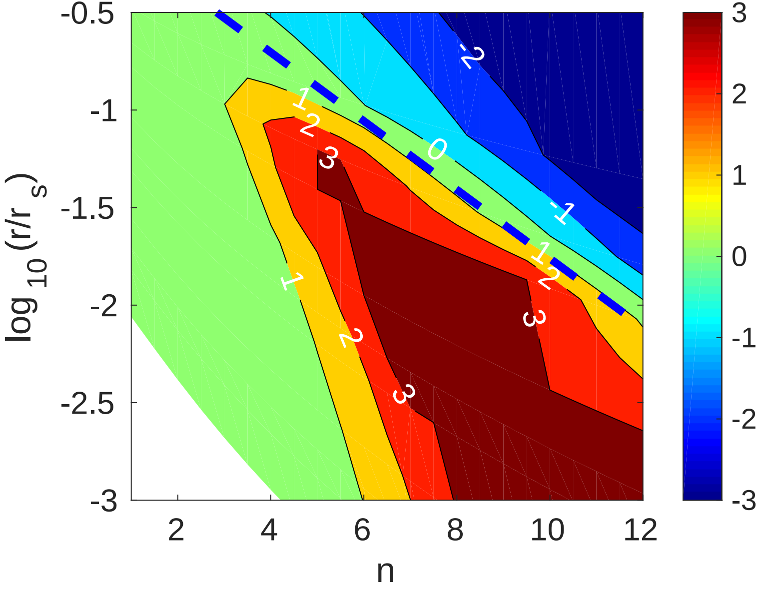
The Fokker-Planck equation has an attractor solution $\rho \propto r^{-\beta}$, where $\beta \approx 1$ (Evans & Collett 1997, Baushev 2015)

$$\frac{df}{dt} = 0 \qquad \text{vs} \qquad \frac{df}{dt} = \frac{\partial^2 [B_{\alpha\beta} f]}{\partial p_\alpha \partial p_\beta}$$

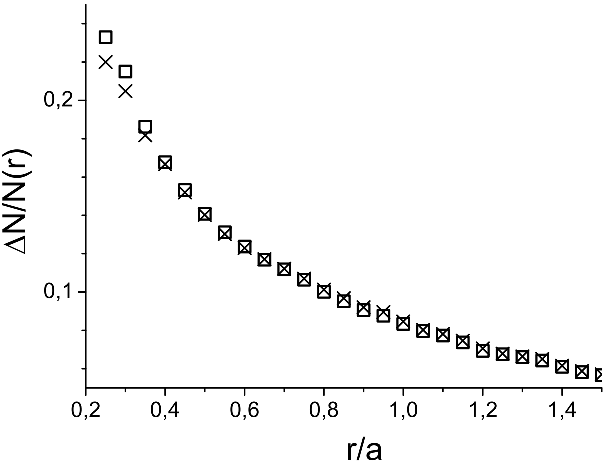
Einasto profile (Navarro et al. 2010)



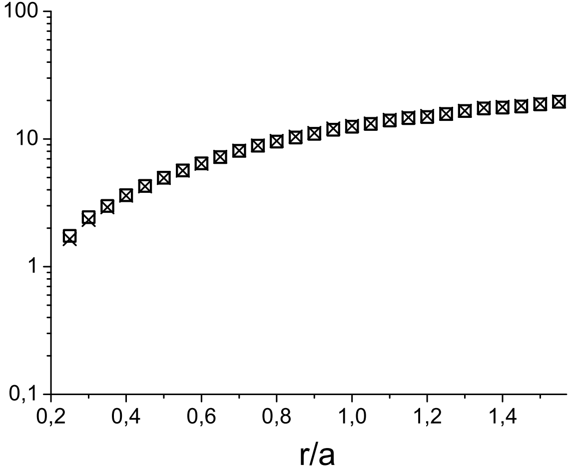
Einasto profile



The upward $\Delta N_+(r)/\Delta t$ (squares) and downward $\Delta N_-(r)/\Delta t$ (crosses) Fokker-Planck streams



$1.7\tau_r \frac{\Delta N_+(r)}{N(r)\Delta t}$ (squares) and $1.7\tau_r \frac{\Delta N_-(r)}{N(r)\Delta t}$ (crosses)



Conclusions

1) Though the cuspy profile is stable, all integrals of motion characterizing individual particles suffer strong unphysical variations along the whole halo, revealing an effective interaction between the test bodies.

2) This result casts doubts on the reliability of the velocity distribution function obtained in the simulations.

3) We find unphysical Fokker-Planck streams of particles in the cusp region. The same streams should appear in cosmological N-body simulations, being strong enough to change the shape of the cusp or even to create it.

4) A much better understanding of the N-body simulation convergency is necessary before a 'core-cusp problem' can properly be used to question the validity of the CDM model.