ICSSNP,
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Relativistic effects in light nuclei and nuclear reactions

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• Three-nucleon system

Three-body Schrödinger equation

$$\left(\frac{p_1^2}{2m}+\frac{p_2^2}{2m}+\frac{p_3^2}{2m}+V_{12}+V_{13}+V_{23}\right)\Psi(1,2,3)=E\Psi(1,2,3)$$

– Good for the bound states

However:

• Three-body scattering

For a given energy above three-body threshold, one can find both nd and nnp systems:

$$nd \Rightarrow \begin{cases} nd & \leftarrow \text{elastic scattering} \\ nnp & \leftarrow \text{breakup} \end{cases}$$

For a many-body system, in the continuous spectrum, the energy does not determine the solution unambiguously. One should add the boundary conditions (Faddeev)!

For nd state it is the product of $\Psi_d \times$ (outgoing nd wave).
For nnp state it is outgoing three-body nnp wave.

• Faddeev equations

3-body scattering amplitude T_{123}

$$T_{123} = T_{12} + T_{23} + T_{31}$$

Momentum space equations:

$$\begin{aligned} T_{12} &= t_{12} + t_{12}\Pi[T_{31} + T_{23}] \\ T_{23} &= t_{23} + t_{23}\Pi[T_{12} + T_{31}] \\ T_{31} &= t_{31} + t_{31}\Pi[T_{23} + T_{12}] \end{aligned}$$

Input: 2-body scattering amplitudes t_{12}, t_{23}, t_{31}.

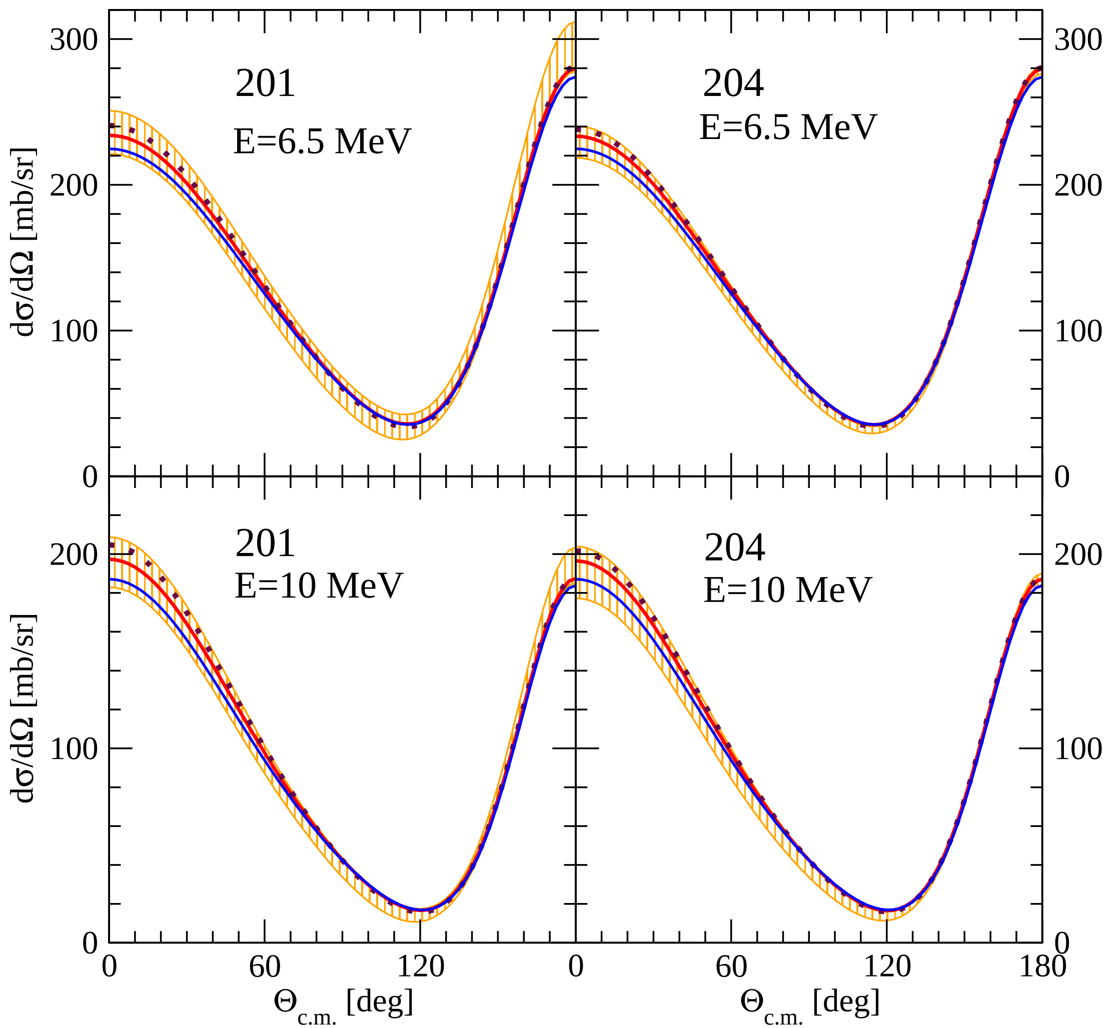
Output: 3-body scattering amplitude T_{123}.

• Working groups

- J. Carbonell (Orsay), R. Lazauskas (Strasbourg) Coordinate space
- Kievsky et al. (Piza)
- H. Witala et al. (Kraków)
- W. Polyzou (Iowa), Ch. Elster (Ohio), W. Glöckle (Bochum)
- A.C. Fonseca et al. (Lisbon)

• Elastic nd scattering

J. Golak et al.



The nd elastic scattering angular distributions at $E_{\mathrm{lab,\,n}} = 6.5$ MeV and 10 MeV

Huge supercomputer calculations!

• Polarized nd elastic scattering.

Definition of A_y

Cross section

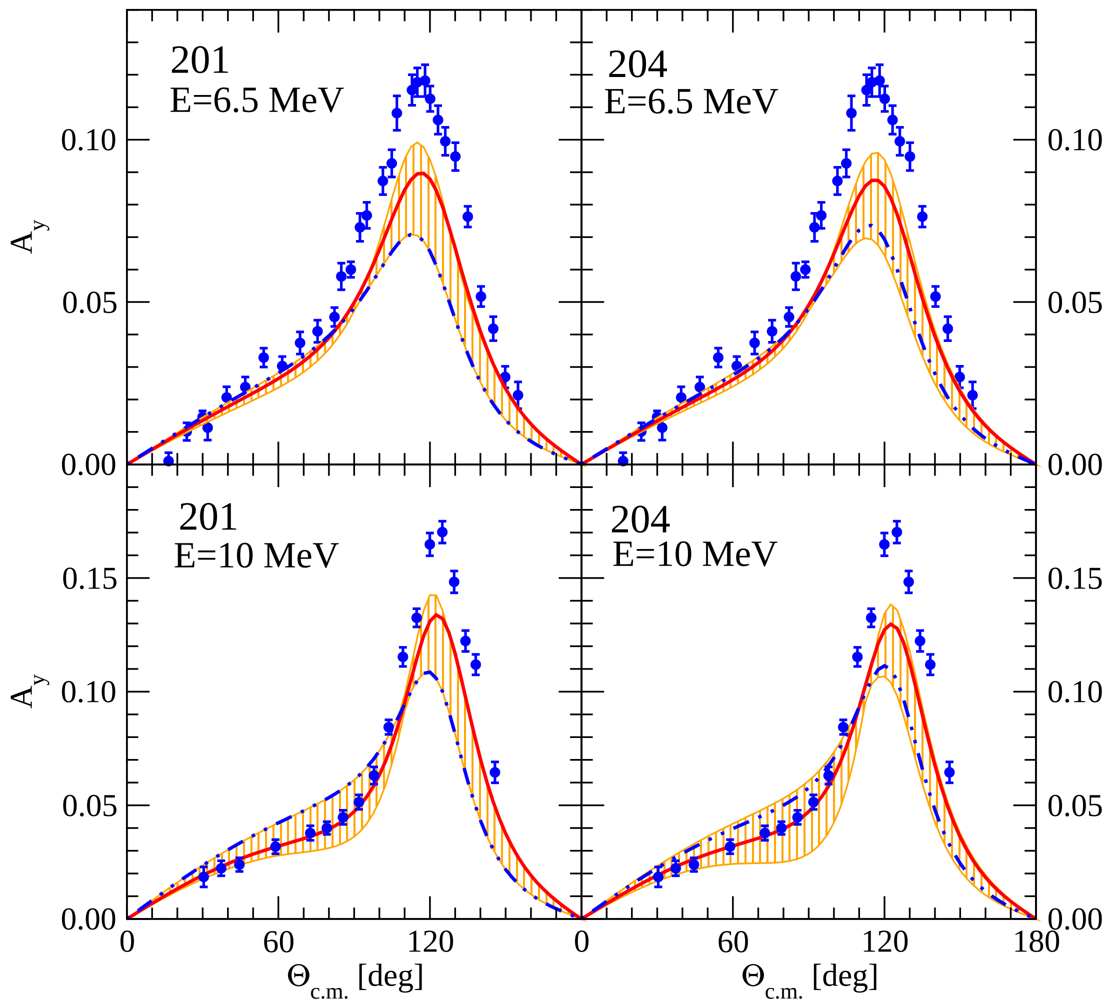
$$\frac{d\sigma}{d\Omega} = A + \vec{S} \cdot [\vec{p} \times \vec{p'}] \; B$$

$$L = Y_L^{(1)} / Y_L^{(2)}, \quad R = Y_R^{(1)} / Y_R^{(2)}$$

$$A_y = \frac{2(L-1)(R-1)}{(L-R)}$$

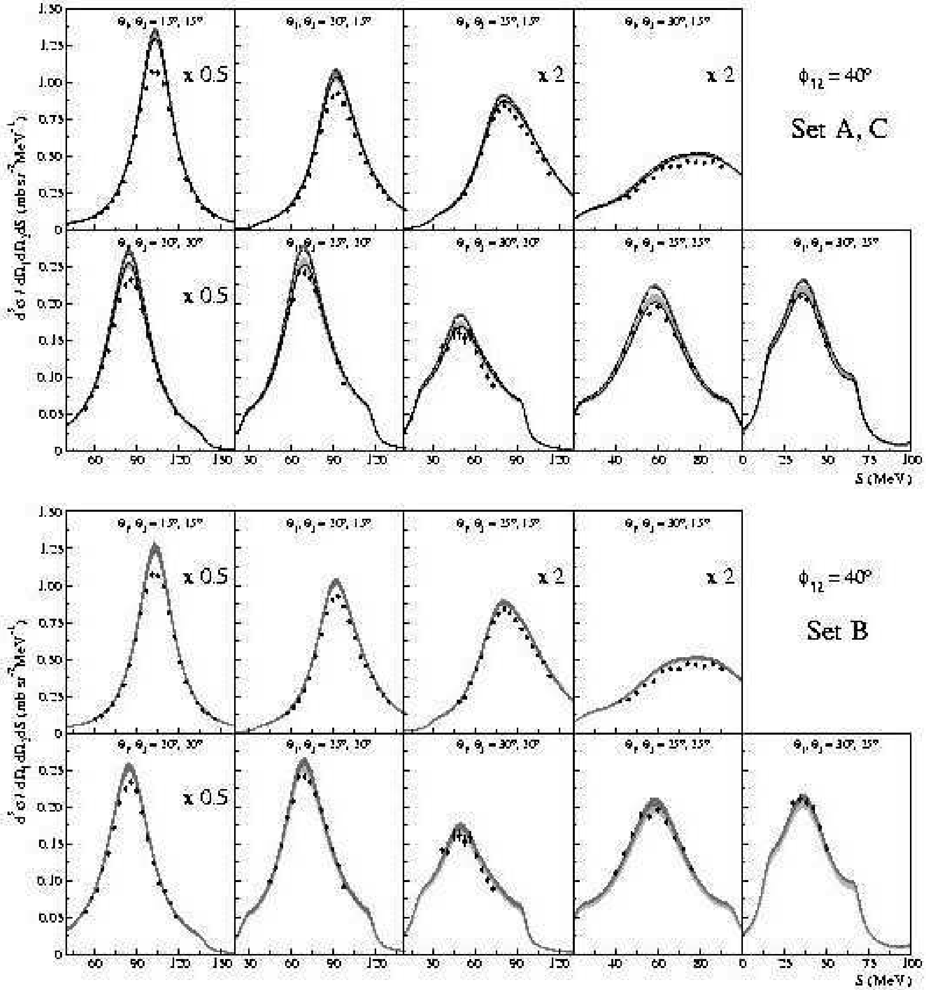
• Polarized nd elastic scattering.

J. Golak et al.



The nd analyzing power. A_y puzzle.

• pd breakup



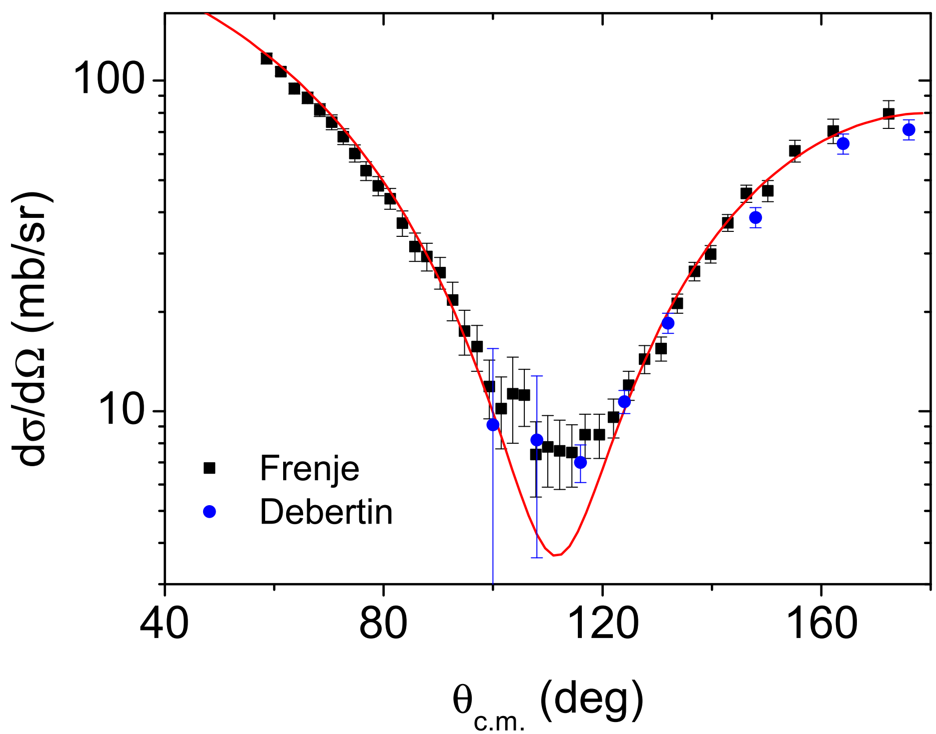
The pd inelastic (breakup) scattering at 130 MeV

• Four-body scattering

Faddeev-Yakubovsky equations

Elastic $n\,^3H$ scattering for 14.4 MeV

J. Carbonell, A. Deltuva, A. C. Fonseca, R. Lazauskas



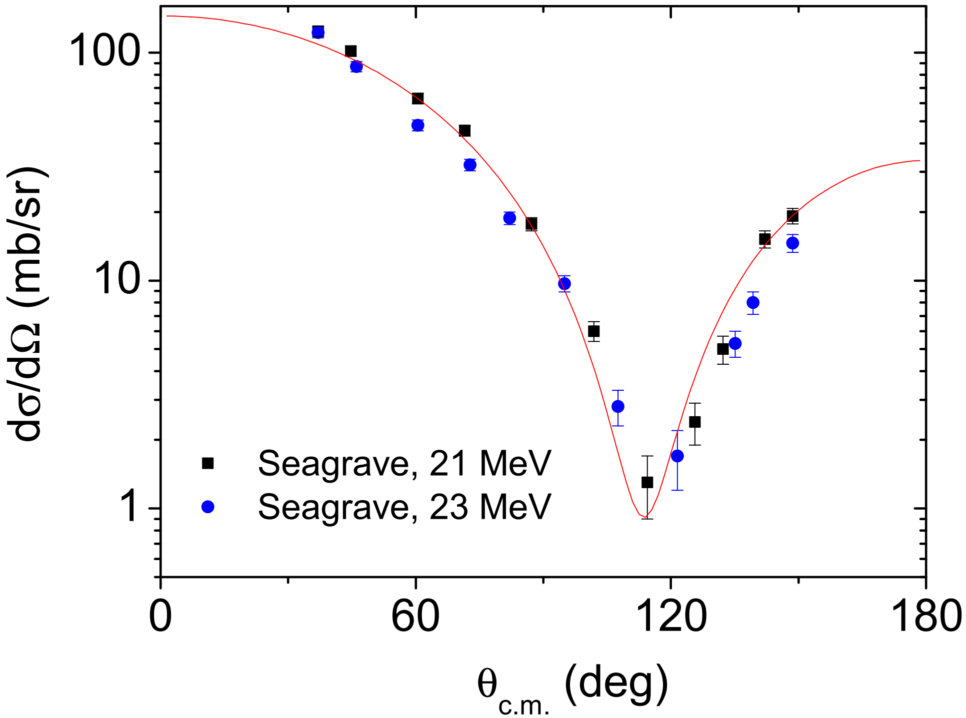
$n + \,^3H$ elastic differential cross-section

for neutrons of lab. energy 14.4 MeV

Elastic $n\,^3H$ scattering for 22 MeV

J. Carbonell, A. Deltuva, A. C. Fonseca, R. Lazauskas

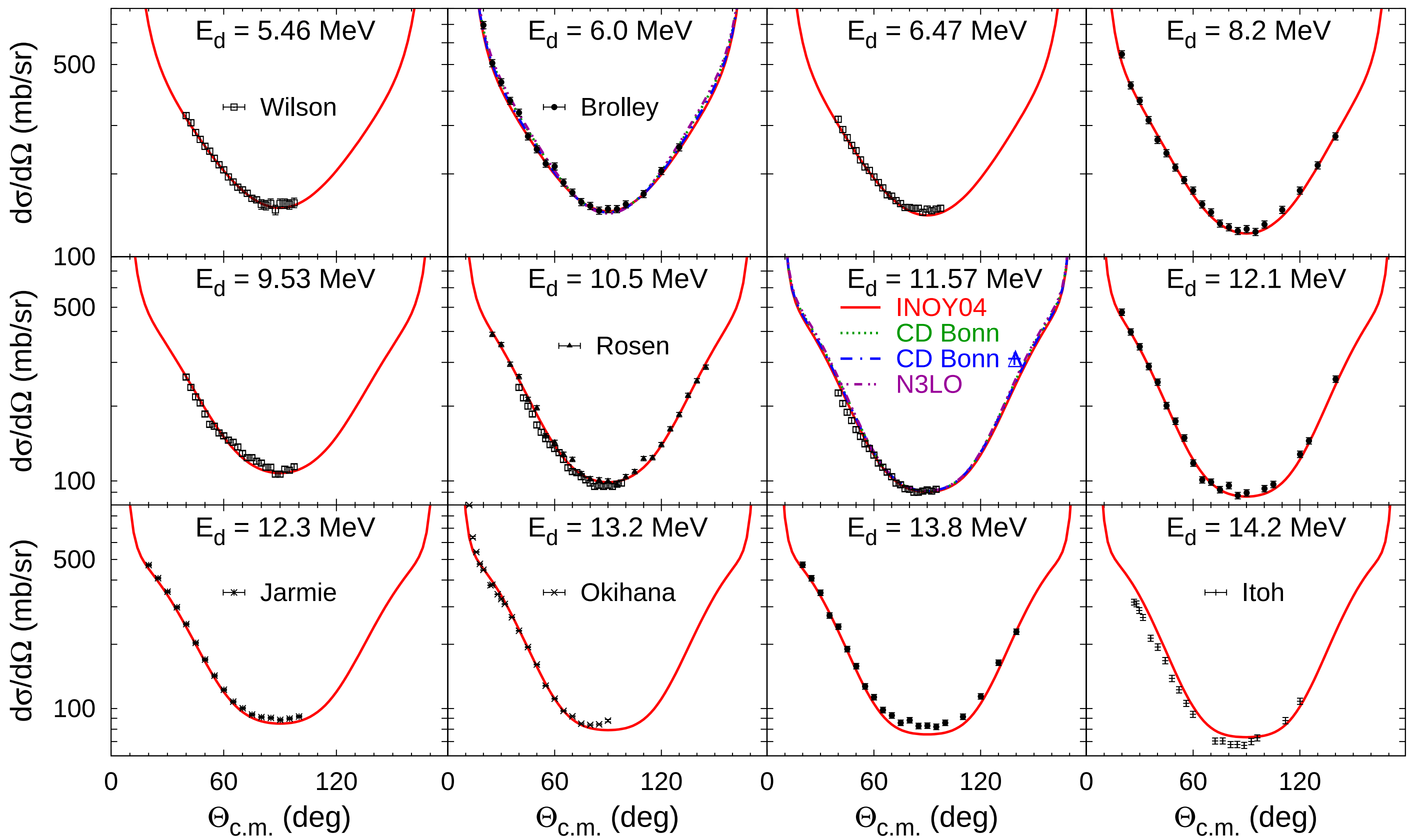
Faddeev-Yakubovsky equations



$n + {}^3H$ elastic differential cross-sections

• Elastic *dd* scattering

A. Deltuva, A. C. Fonseca



Differential cross section of $d+d$ elastic scattering as a function of c.m. scattering angle at deuteron beam energies ranging from 5.46 to 14.2 MeV.

• Analysis of particular singularities

$\Rightarrow$ full solution

I.S. Shapiro,
Theory of direct nuclear reactions
based on analysis of singularities of amplitudes
(pole singularities, the singularities of the triangle graphs).

Solutions of the Faddeev equations contain all these singularities (and not only them).
They provide full amplitudes with all its infinite number of singularities!

Outstanding progress has been achieved!

• Relativistic effects

Two sources of relativity:

1. Fast internal motion of nucleons in nuclei
2. High energy beam

Preliminary question

Binding energy of nucleus is small
(relative to the nucleon mass).

Can system with small binding energy be relativistic?

The answer: yes!

• Non-relativistic v.s. relativistic

Non-relativistic kernel:

$$V_{non-rel.}(r) = -\frac{\alpha}{r}e^{-\mu r} \Rightarrow V_{non-rel.}(\vec{q}) = \frac{-4\pi\alpha}{\vec{q}^2 + \mu^2}$$

Solve the Shrödinger equation.

Relativistic kernel:

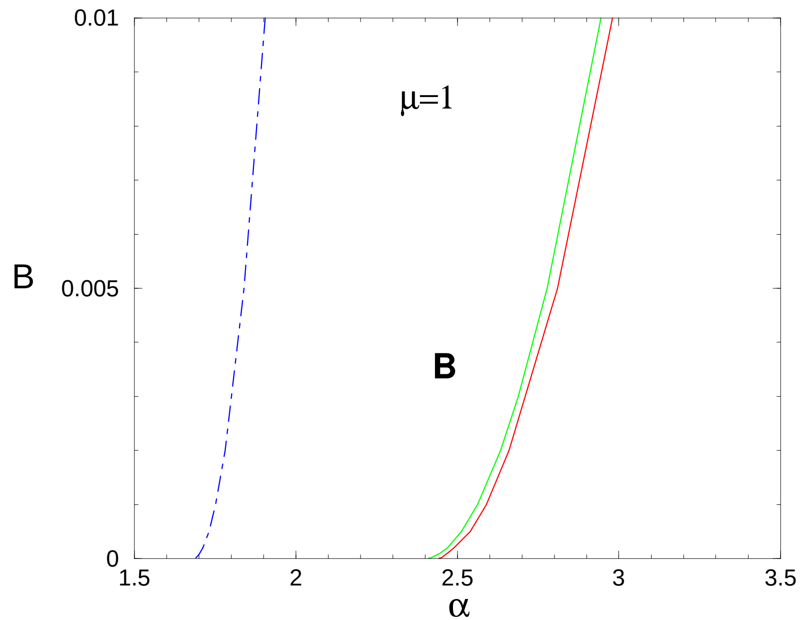
$$V_{rel.}(q) = \frac{-4\pi\alpha}{\vec{q}^2 - q_0^2 + \mu^2}$$

Solve Bethe-Salpeter and light-front equations.

Compare non-relativistic and relativistic results.

• Comparison

M. Mangin-Brinet and J. Carbonell, Phys. Lett., **B474** *(2000) 237.*

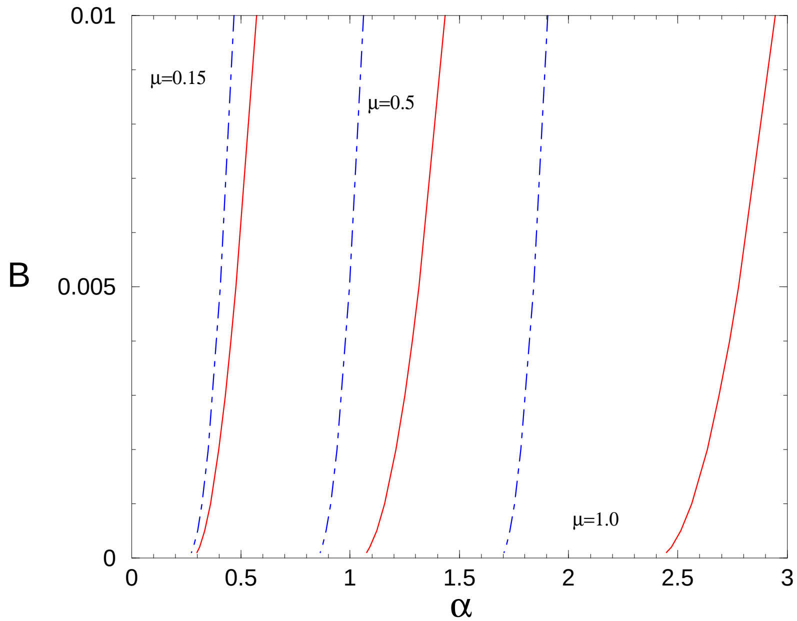


Two-body binding energy B versus coupling constant α. $\mu = 1$

$$B = 0,\ \mu = 1 \Rightarrow \alpha_{NR} = 1.7, \quad \alpha_{rel} = 2.4$$

- large difference!

Dependence on the exchange mass μ



o-body binding energy B versus coupling constant α. $\mu = 1,\ 0.5,\ 0.15$

The relativistic effects decrease when μ decreases.

True nonrelativistic systems - are the systems with small binding energy and Coulomb interaction.

• Remarks

- In OBE-inspired potentials, the ω- and ρ-mesons are heavy mesons:

$$m_\omega \approx m_\rho \approx 0.8\, m_N$$

Therefore the relativistic effects in nuclei should be visible even at low-energy.

- If the system is mainly non-relativistic (average momenta are small), the relativistic tail in momentum distribution always exists. It determines, for example, e.m. form factors at high momentum transfer.

Therefore, at large Q^2 the relativistic effects in e.m. form factors are always important.

How to describe a relativistic few-body system?

"How to describe" means:

What theoretical approach should be applied?

– Big ambiguity

• State vector $|p\rangle$

For deuteron (for example):

$$|p\rangle = \begin{pmatrix} pn \\ pn\pi \\ pn\pi\pi \\ pn\pi\pi\pi \\ \dots \end{pmatrix} = \begin{pmatrix} \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \\ \dots \end{pmatrix}$$

$|p\rangle$ satisfies the Schrödinger equation:

$$i\partial|p\rangle/\partial t = H|p\rangle$$

State vector $|p\rangle$ is defined (in 4-dim. space) on the plane

$$t = const \quad (= 0)$$

$\psi_2,\ \psi_3, \dots$ are the Fock components of the state vector.

• Light-front dynamics (LFD)

Dirac (1949)

State vector is defined not on the plane

$$t = 0$$

but on the light-front plane

$$t + z = 0$$

$z = \pm ct$ is the equation of the light front in the space-time.

• Two approaches to relativistic few-body physics

1. To study the Fock components of the state vector $|p\rangle$ on the LF plane.
2. Instead of the state vector $|p\rangle$, consider Bethe-Salpeter amplitude (1951):

$$\Phi(x_1, x_2, p) = \langle 0|T\Big(\varphi(x_1)\varphi(x_2)\Big)|p\rangle$$

• Relativistic deuteron wf

- Non-relativistic: 2 spin components (S and D waves)
- Gross (one particle on mass-shell: 4 spin components
- Light-front: 6 spin components
- Bethe-Salpeter: 8 spin components

• Relativistic deuteron LF wf

LF: $\omega\cdot x = 0$, $\vec{n} = \vec{\omega}/|\vec{\omega}|$.

$$
\begin{aligned}
\vec{\psi}(\vec{k},\vec{n}) &= f_1\frac{1}{\sqrt{2}}\vec{\sigma} + f_2\frac{1}{2}\left(\frac{3\vec{k}(\vec{k}\cdot\vec{\sigma})}{\vec{k}^2} - \vec{\sigma}\right) \\
&+ f_3\frac{1}{2}(3\vec{n}(\vec{n}\cdot\vec{\sigma}) - \vec{\sigma}) \\
&+ f_4\frac{1}{2k}(3\vec{k}(\vec{n}\cdot\vec{\sigma})3\vec{n}(\vec{k}\cdot\vec{\sigma}) - 2(\vec{k}\cdot\vec{n})\vec{\sigma}) \\
&+ f_5\sqrt{\frac{3}{2}}\frac{i}{k}[\vec{k}\times\vec{n}] \\
&+ f_6\frac{\sqrt{3}}{2k}[[\vec{k}\times\vec{n}]\times\vec{\sigma}]
\end{aligned}
$$

V.A.K., Nucl. Phys. A **362**, 331 (1980).

• E.m. structure of deuteron

Nucleon: two form factors

$F_C(Q^2)$ – charge form factor;
$F_M(Q^2)$ – magnetic form factor.

Deuteron: three form factors

$F_C(Q^2)$ – charge form factor;
$F_M(Q^2)$ – magnetic form factor;
$F_Q(Q^2)$ – quadrupole form factor.

• ed cross section

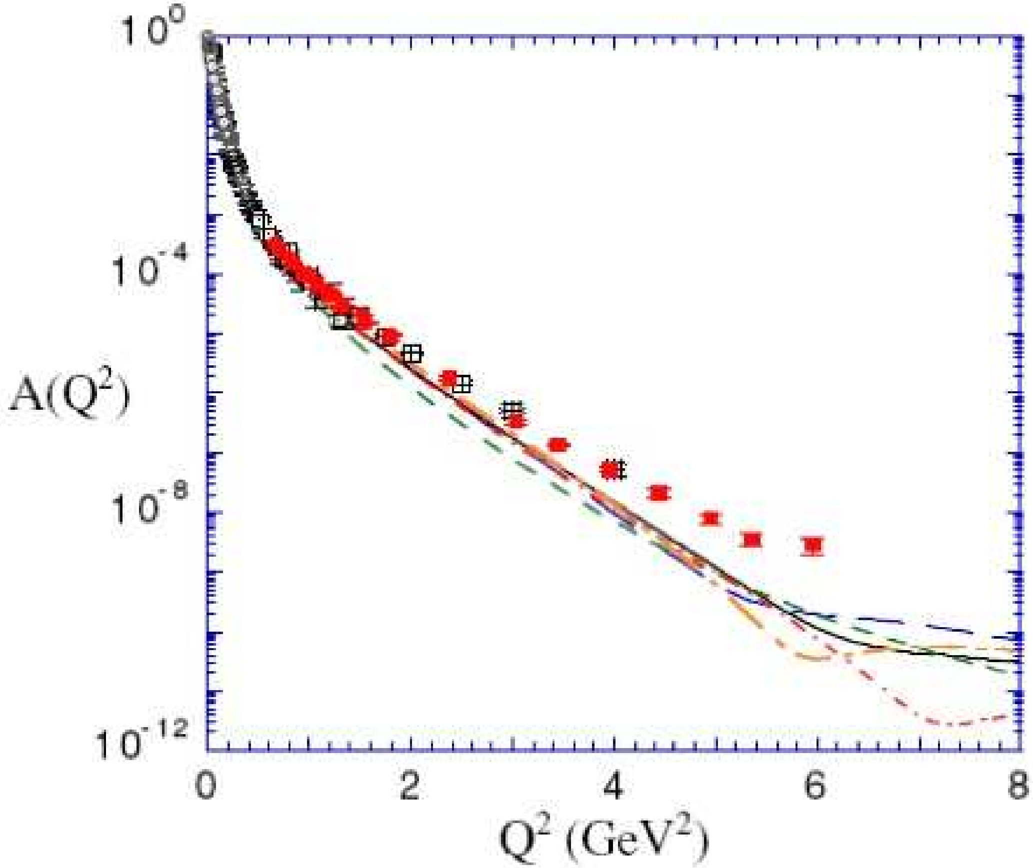
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left(A(q^2) + \tan^2\frac{1}{2}\theta\ B(q^2)\right)\ ,$$

$$A(q^2) = F_C^2(q^2) + \frac{8}{9}\eta^2 F_Q^2(q^2) + \frac{2}{3}\eta F_M^2(q^2)\ ,$$

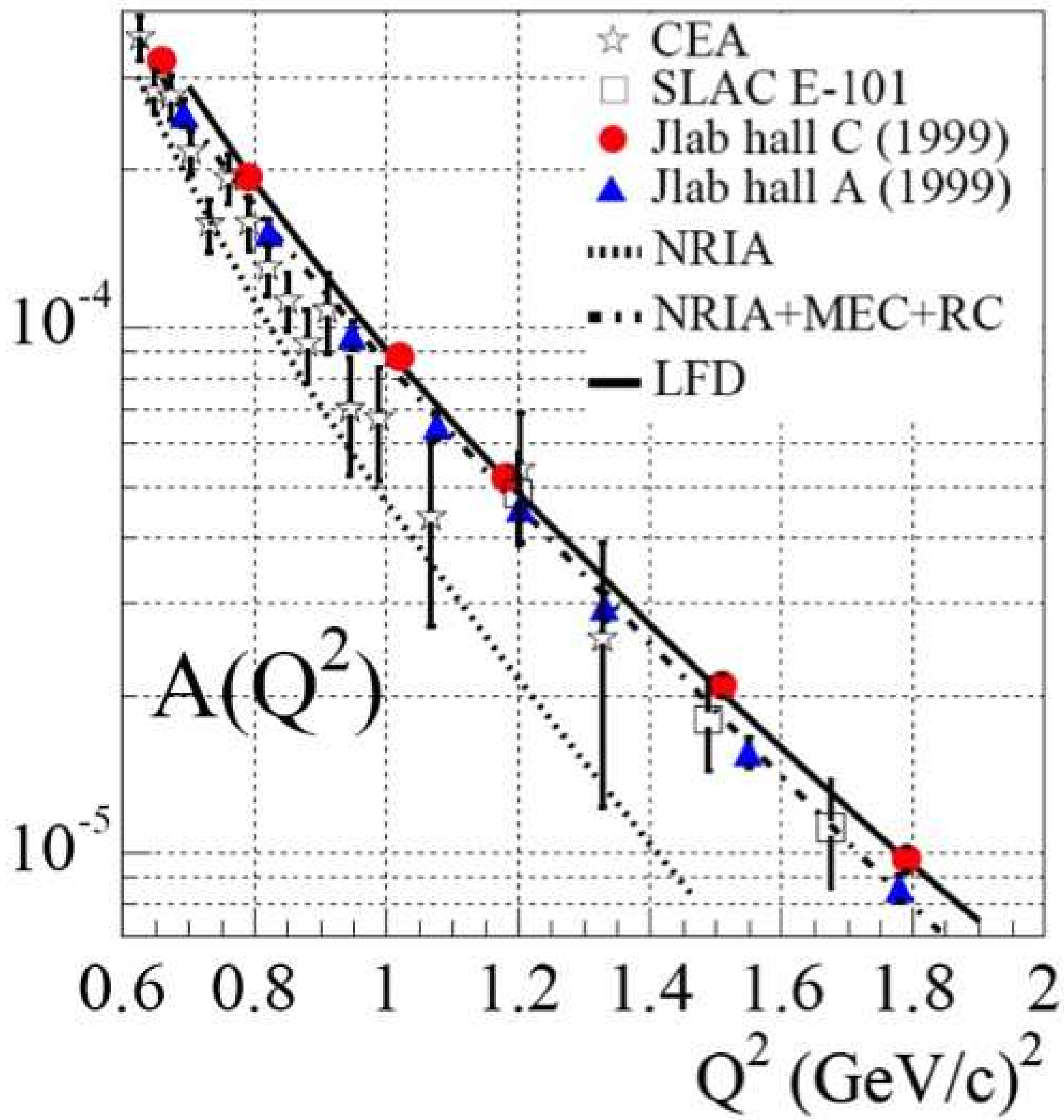
$$B(q^2) = \frac{4}{3}\eta(1+\eta)F_M^2(q^2)\ .$$

$$\eta = \frac{Q^2}{4M^2}$$

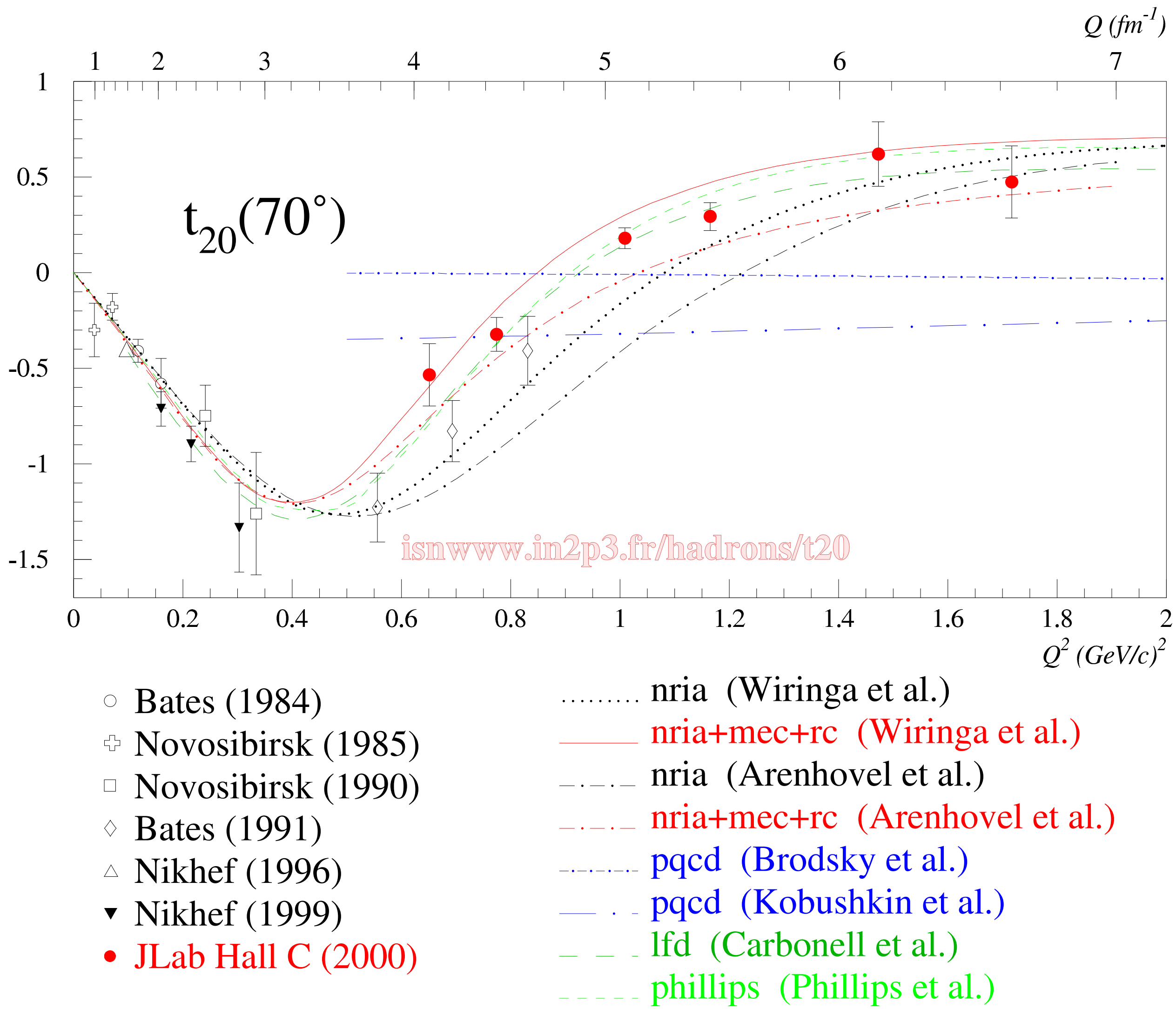
• Structure function A, non-rel. calc.



Structure function $A, Q^2 \leq 2\,(GeV/c)^2$



• Polarization observable t_{20}



• Three-nucleon systems

Underbinding of tritium

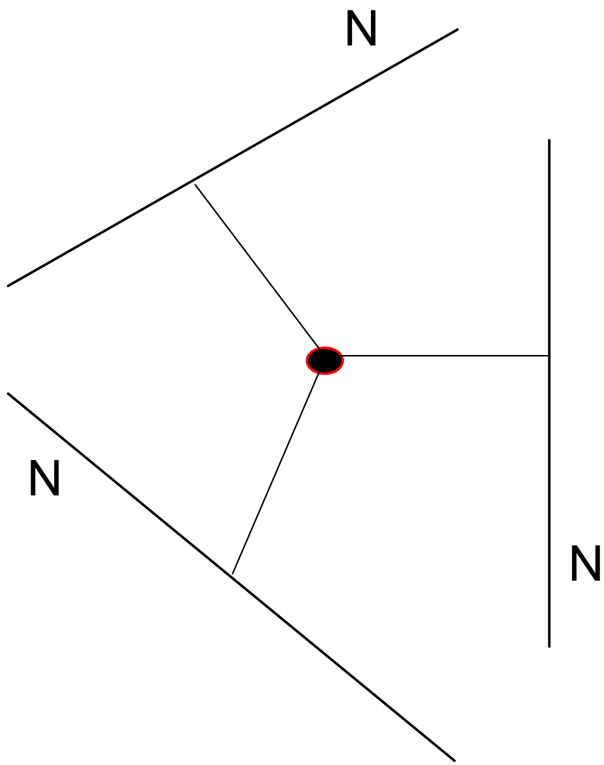
Paris:	E = -7.38 MeV
RSC:	E = -7.23 MeV
Experiment:	E = -8.48 MeV

– Removed by three-body forces.

$V_{2-body} = V(\vec{r}_1 - \vec{r}_2)$; $V_{3-body} = V(\vec{r}_1 - \vec{r}_2, \vec{r}_1 - \vec{r}_3, \vec{r}_2 - \vec{r}_3)$.

– However, the three-body forces are partially a consequence of relativity.

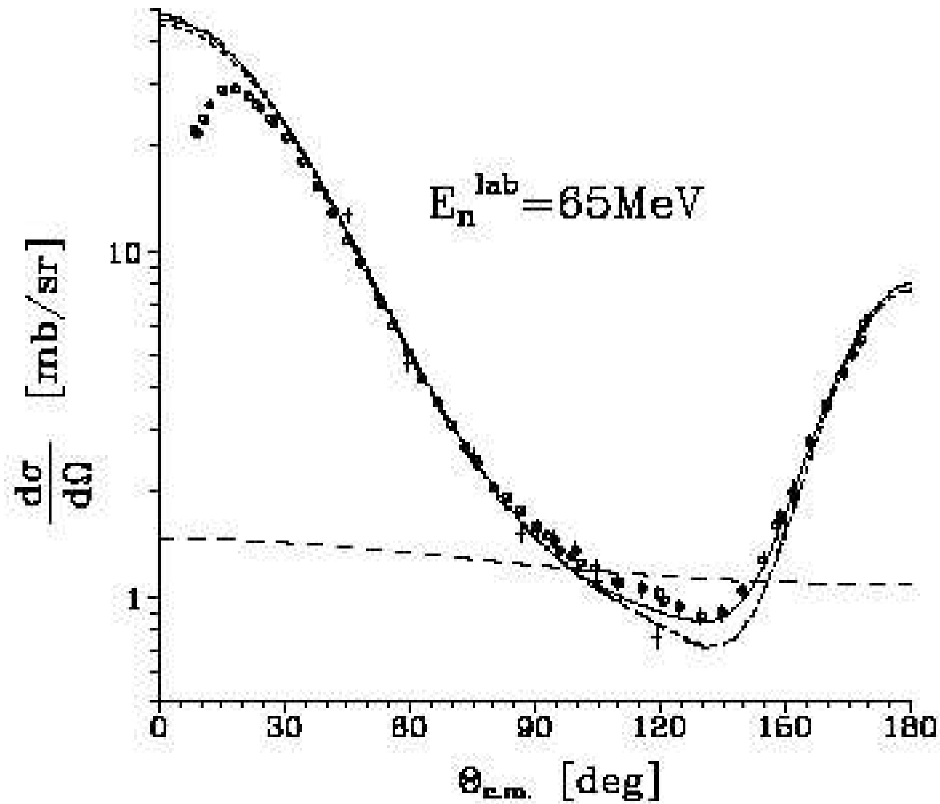
"Intrinsic" three-body forces can also contribute.



In many-body systems, relativistic effects generate three-body (four-body,... many-body) forces.

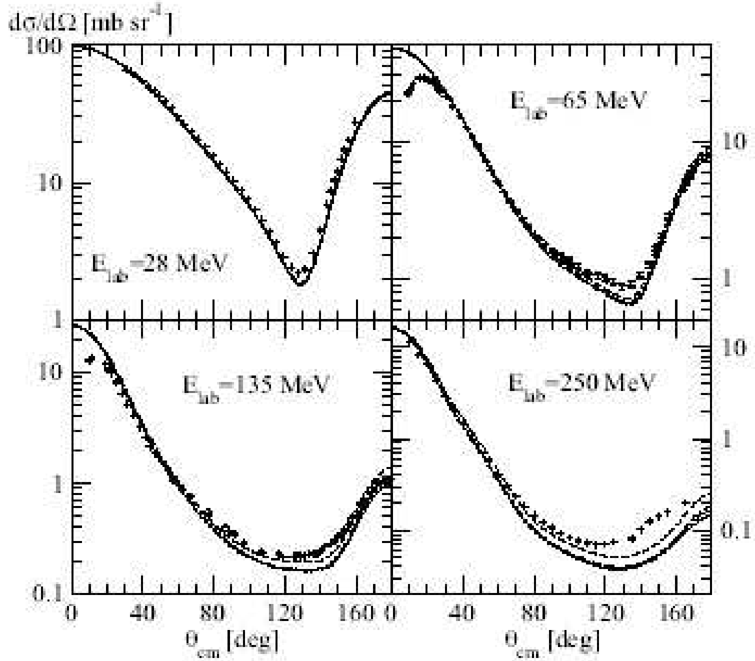
All that should be taken into account simultaneously.

• pd elastic scattering. 3-body forces



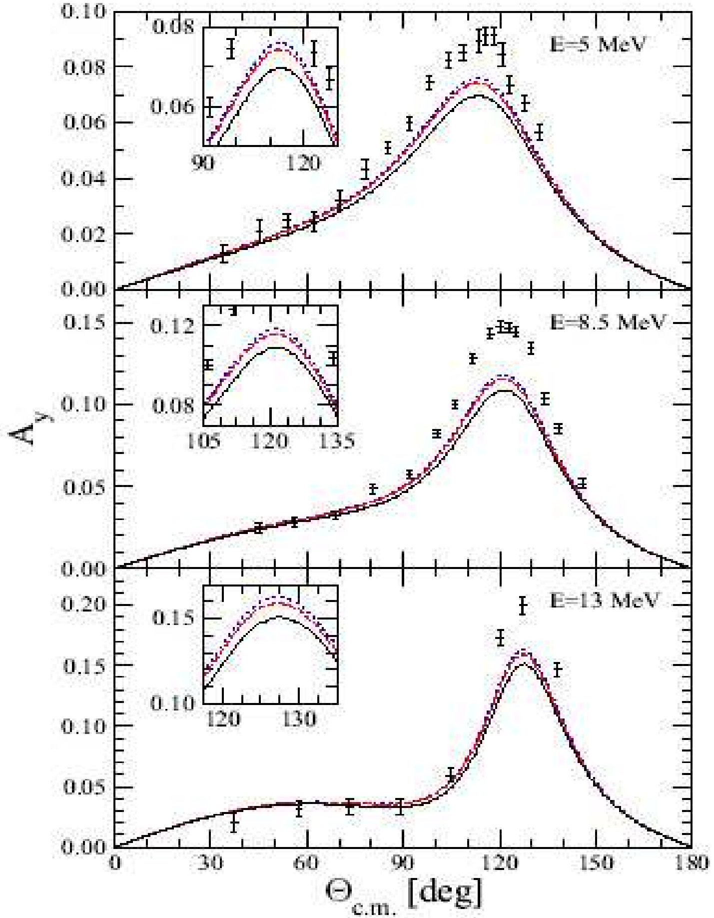
Short dashed line – non-relativistic.
Solid line – non-relativistic + 3-body forces.

• pd elastic scatt. Relativistic effects.



Solid line – non-relativistic. Dashed line – relativistic.

pd elastic scatt. Analyzing power A_y



Dotted line – non-relativistic.
Solid line – relativistic.

• Comment

There are more things in heaven and earth, Horatio,

Than are dreamt of in your philosophy.

William Shakespeare.

The Tragedy Of Hamlet, Prince Of Denmark

Есть многое в природе, друг Горацио,
Что и не снилось нашим мудрецам.

Relativistic effects + three-body forces might be responsible for these *”more things”*.

• Conclusions

- Light nuclei and reactions with them are well understood from the "first principles".
- Their description requires incorporation of relativistic effects.
- However, there are still some discrepancies to be overcome.