Gluon correlators and effective gluon mass in lattice QCD

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Session-Conference SNP PSD RAS,
06.06-08.06.2017, Nal'chik

Abstract

We make new simulations of $SU(2)$ zero spatial momentum (ZM) gluon correlator paying special attention to possible lattice artefacts and Gribov copy effect. In particular we started investigation of the correlator dependence on the choice of boundary conditions by comparing results for periodic and zero-field (ZF) boundary conditions at various β values. Time behaviour of the correlators (at least for ZF b.c.) corresponds to constant in time effective gluon mass thus providing additional evidence in favour of decoupling behaviour of momentum-dependent gluon propagator in the IR region. We have found that at fixed lattice sizes and β values the ZM gluon correlator for periodic and ZF boundary conditions can differ considerably.

Introduction

- In nonperturbative studies of Landau gauge gluon and ghost propagators

$$D^{ab}_{\mu\nu} = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{Z(q^2)}{q^2}, \;\; G^{ab} = \delta^{ab}\frac{J(q^2)}{q^2}$$

  with continuum Dyson-Schwinger (DS) or Funct. Renorm. Group (FRG) Eqs. and within the lattice approach

  2) decoupling, or regular solution has been found [Boucaud et al. '05 - '07; Aguilar et al. '04 - '08, lattice simulations '06-09'] $D(q^2) \to D_0 = const, \;\; J(q^2) \to J_0 = const$

- Decoupling solution is the only one found in lattice simulations with periodic boundary conditions (PBCs). It describes "effectively massive" gluon and the BRST symmetry is broken on it. To exclude possibility of simulation artefacts thorough investigations of finite volume, finite size and Gribov copy effects have been undertaken under PBCs.

- However dependence of the lattice propagators on choice of boundary conditions has not been reported, to our knowledge.

Motivation

- To start consideration of this issue we turn to simulations of zero spatial momentum gluon correlator, which has been studied first by Mandula and Ogilvie '1987' under PBCs in $SU(3)$ gluodynamics and later by other groups (e.g.,Gupta et al '87', Bernard et al '93').

- From decay of their gluon data the hypothesis of nonzero gluon mass $m(t)$ arises, but their "effective masses"$m(t)$ differ considerably from being constant in t. It can be due either to Gribov ambiguity, or to the choice of PBCs (or to both).

- Gribov copy problem for lattice gauge fixing has found good practical solutions by means of using the Simulation Annealing (SA) method (Bogolubsky et al '06-09').

- Investigation of alternative BCs can prove to be of importance for investigation of propagators and even lead to qualitatively new results, because PBCs introduce some apriori restrictions on possible solutions.

- An interesting question is: how could conclusions on "massiveness" of gluon change when other types of BCs are used.

Gauge fixing: SA+OR

In order to fix the Landau gauge we apply a gauge transformation $g(x)$ to link variables $U_{x,\mu} \in SU(3)$ or $SU(2)$) such that the gauge functional is maximized

$$F_U[g] = \sum_{x,\mu} \frac{1}{N_c} \,\mathfrak{Re}\,\mathrm{Tr}\, {}^{g}U_{x,\mu} \,.$$

⇒ For $A_\mu(x+\hat{\mu}/2) := (1/2ig_0)\left(U_{x,\mu} - U^\dagger_{x,\mu}\right)_{\mathrm{traceless}}$
this is equivalent to $\Delta_\mu A_\mu = 0\,,$

⇒ but not unique: Gribov copies (local extrema)

⇒ search for global extrema -

for this search we use Simulated Annealing (SA), finalized by Overrelaxation (OR) steps.

- SA is a "stochastic optimization method" – here with the statistical weight $W[g] \propto \exp\{F_U[g]/T\}$ –
  allowing quasi-equilibrium tunneling through functional barriers, in the course of a "temperature" T decrease.

- In principle - with infinitely slow cooling down - it allows to reach global extrema.

ZM gluon correlator, ZF BCs

by applying very long SA procedure followed by OR local extremization. This allows us to reach the FMR region, or, in other words, to reach regionWe study behaviour in Euclidean time of zero spatial momentum correlator

$$S(t) = \sum_{\vec{x}} \sum_{i=1,2,3} \text{Tr } \langle A_i(t,\vec{x}) A_i(0,\vec{x}) \rangle$$

and of the correlator $T(t)$

$$T(t) = \sum_{\vec{x}} \text{Tr } \langle A_0(t,\vec{x}) A_0(0,\vec{x}) \rangle$$

in the Landau gauge on lattices $L_s^3 * L_t, L_t = 2L_s$ (mainly), with $L_s = 10, 12, 14, 16, 18$; our maximal lattice extension was $22^3 * 30$. In the first series of simulations we use zero-field boundary conditions (ZF BCs), and make high-statistics Monte-Carlo simulations on lattices $L_s^3 * (2L_s)$ with various L_s. The typical number of MC configurations was of order 10^4 with one gauge fixed copy obtained for each MC configurations

being very close to global extremum of the gauge functional G_F. The correlator function $T(t)$ is constant in t with very high accuracy, which can show reliability of the results. The varying correlator function S(t) is plotted in Fig. 1.

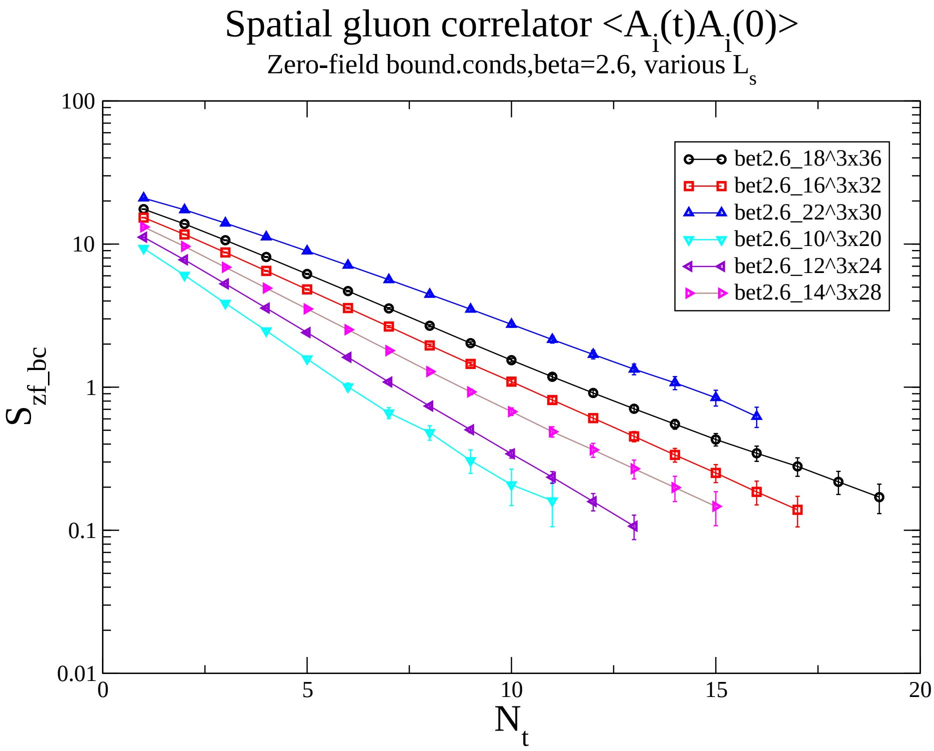

Figure 1: The ZM gluon propagator $S(t)$ for $\beta = 2.6$, ZF BCs and various lattice sizes .

- Note that for all lattice extensions dependence $ln\ S(t)$ is close to linear, resulting in constant in time t “effective mass”.

ZM gluon correlator, ZFBC vs PBC

- The question is: whether change of boundary conditions type can lead to essential/qualitative difference of ZM gluon correlator?

We simulate ZM $S(t)$ on lattice $L_s^3 * L_t, L_t = 2 * L_s$ with $L_s = 12$ both for periodic BCs (at $\beta = 2.2, 2.3, 2.4, 2.5, 2.6$) and zero-field BCs (at $\beta = 2.2, 2.3, 2.4, 2.5$). Surprisingly enough, the results can differ considerably, see Fig.2.

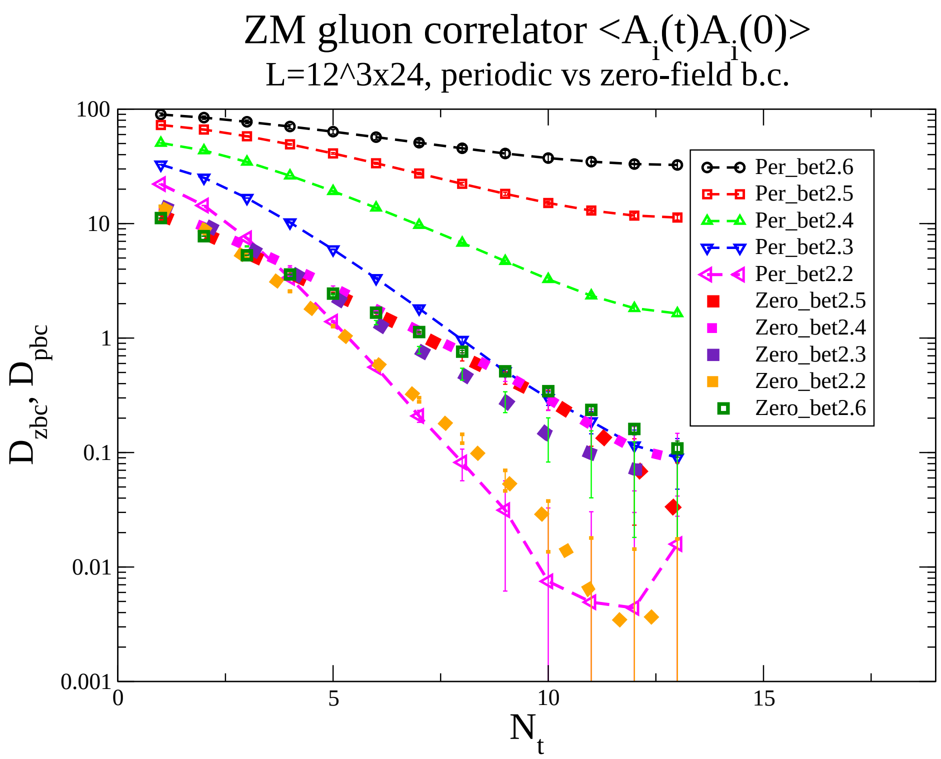

Figure 2: Comparison of the ZM gluon correlator $S(t)$ for zero-field and periodic BCs, $L_s = 12$ and various β.

- Note that difference of correlator $S(t)$ for periodic and zero-field BCs grows with β increase, becoming considerable at $\beta = 2.5$.

- One can see exponential decay of $S(t)$ for both types of boudary conditions.

Effective mass, ZFBC vs PBC

By fitting curves for correlators $S(t)$ we extract gluon effective masses $m(t)$

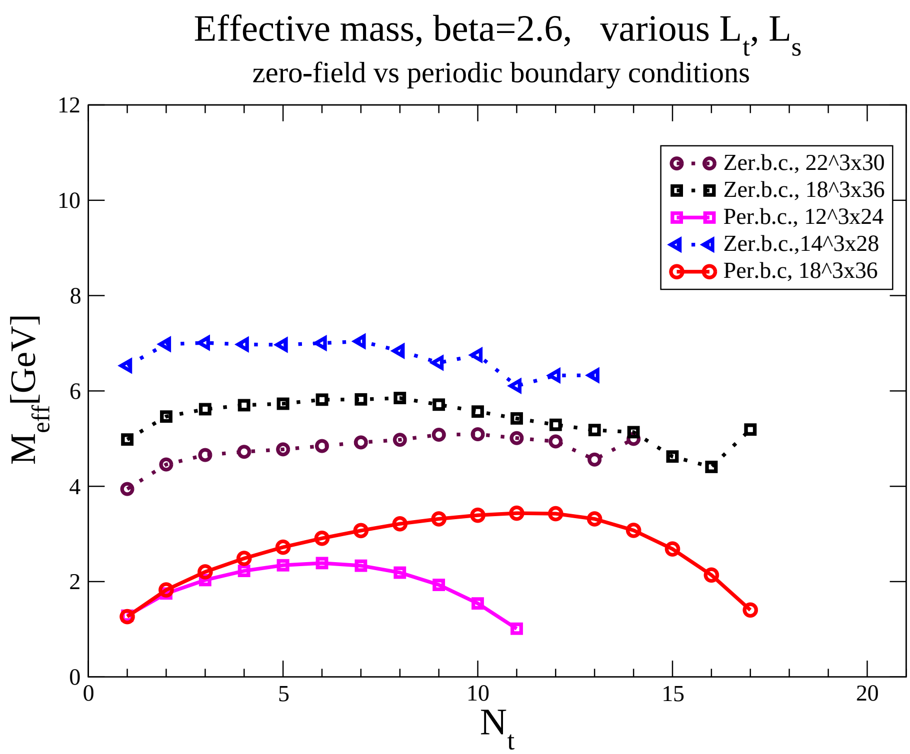

Figure 3: Comparison of effective masses $m(t)$ for zero-field and periodic BCs, for various L_s and L_t at $\beta = 2.6$.

- One can see that $m(t)$ curves for both types of boudary conditions get closer with growing of lattice sizes L_s and L_t. For both types of BCs effective gluon mass is nonzero!

Conclusions and Questions

- Dependence of Green functions/correlators in lattice gauge theories on choice of boundary conditions and Gribov copies needs careful study, in particular, by computer simulations.

- High-statistics MC lattice study at zero-field boundary conditions shows exponential decay of zero spatial momentum correlator in $SU(2)$ gluodynamics with "effective mass" very close to being constant.

- Behaviour of propagators/correlators in lattice gauge theories can strongly depend on the choice of boundary conditions.

- Nonzero gluon "effective mass" confirmed both for ZF BCs and PBCs with Gribov problem removed gives additional arguments in favour of nonzero effective mass of momentum-dependent propagator, having been found for "decoupling" solution.

- However it would be interesting to make lattice simulations of momentum-dependent propagator with BCs differing from periodic ones, because it would allow to estimate region of validity of the results earlier obtained for PBCs.

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